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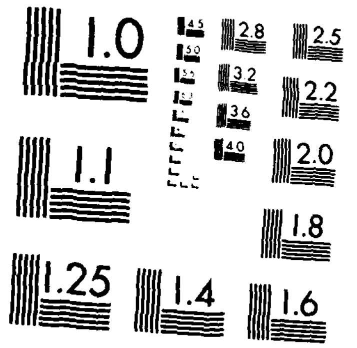

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A FAST HEURISTIC BASED ON SPACEFILLING CURVES FOR MINIMUM-WEIGHT MATCHING IN THE PLANE

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Abstract

~~We~~ The authors present a heuristic to find a good matching on n points in the plane. It is essentially sorting and so runs in $O(n \log n)$ time (worst case) or linear expected time. Its performance is competitive with that of previously suggested methods. However, it has the advantages of being trivial to code and of being indifferent to the choice of metric or the probability distribution from which the points are drawn.

Submitted to Information Processing Letters

Keywords: Approximation algorithms, minimum weight matching, spacefilling curves, travelling salesman problem.

1. Introduction

We consider the problem of finding the minimum-weight matching on n points in the unit square. The interpoint distance may be any norm in R^2 (for example, rectilinear, Euclidean, maximum component). This problem arises in various routing problems, such as in sequencing operations for a mechanical plotter [3]. Although Edmonds' algorithm [5] will construct an optimum matching in $O(n^3)$ time, faster, simpler heuristics are preferred in practice. Consequently this problem has received much recent study. In his survey [1], Avis preferred two algorithms, the strip method of Supowit, Reingold, and Plaisted [8] and the cell order method of Iri, Murota, and Matsui [3]. Both obtain a matching by selecting alternate edges from a good travelling salesman tour. Our approach is an outgrowth of these two methods.

The strip heuristic partitions the square into columns, sorts the points within each column, and visits all the points of one column before proceeding to the next adjacent column. Iri et al. construct a similar scheme called the "serpentine cell order" wherein the square is divided into O(n) cells, the cells are sequenced in strips (Figure 1), and all the points in one cell are visited before proceeding to the next cell. Then an improved scheme, the "spiral rack cell order," is obtained by resequencing the cells in a spiral fashion (Figure 2) so that cells that are close in sequence tend to be close in distance. We carry this improvement further by breaking the square into an infinite number of cells sequenced according to an infinitely crinkly "spacefilling curve" (Figure 3).

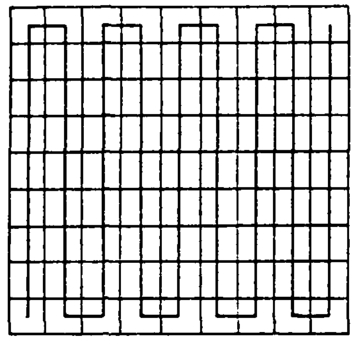


Figure 1: The "serpentine cell order" of Iri, Murota, and Matsui.

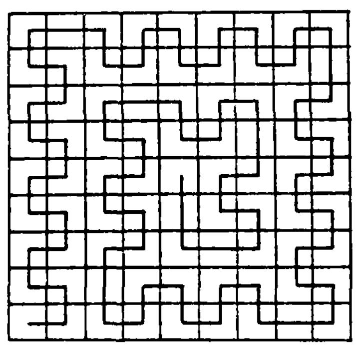
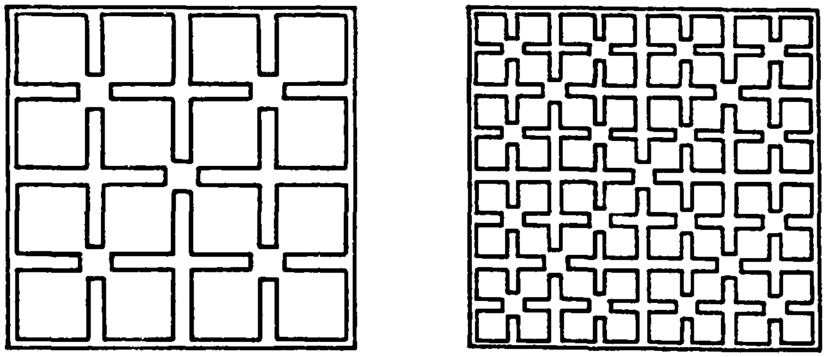


Figure 2: The "spiral rack cell order". It is better than the serpentine cell order since it tends to visit most of the cells in a region before moving to another region.



<u>Figure 3</u>: The second and third approximations, respectively, to a spacefilling curve which is the limit of the sequence. The spacefilling curve (or one of its approximations) is better than the spiral rack cell order because it tends to visit ALL of the points (or cells) in a region before moving to a new region.

2. The algorithm

For each point (x,y) we compute, by an analytic expression, its position θ along a spacefilling curve (a spacefilling curve ψ: θ (x,y) is a continuous mapping from the unit interval onto the unit square. Sorting by θ produces a heuristic travelling salesman tour. To derive a matching, simply pair the first and second points of the tour, then the third and fourth points, and so on. An optional additional step is to construct a second matching by pairing the first and last points, the second and third points, and so on, and to select the better matching.

This description in fact gives a whole class of algorithms, since any spacefilling curve may be used.

3. Computational effort

The algorithm consists essentially of sorting and so requires O(n log n) operations at most. It may also be implemented to run in expected linear time using a randomized algorithm such as BINSORT [4,9]. Alternatively, O(n) cells of the unit square may be sequenced according to the spacefilling curve to give a linear time algorithm as in Iri et al.

For our algorithm, only additions and subtractions are required. However, to perform the additional extra step of choosing the better of two matching requires the evaluation of interpoint distances (for example, square roots for Euclidean distances).

4. Quality of solution

Our performance analysis is for a particular curve, similar to one by Sierpinski [7], that seems well-suited to problems of combinatorial

optimization. The useful properties of this spacefilling curve are outlined in Bartholdi and Platzman [2] and are analyzed in detail in Platzman and Bartholdi [6].

The longest travelling salesman tour produced will be no more than $2\sqrt{n}$ and so this bound certainly holds for the matching. Furthermore, with the additional step, the heuristic matching will be no greater than $\sqrt{n}$.

The worst case ratio of heuristic to optimal matching is $+\infty$ and this is assymptotically achievable (for example, when there are points at every bend in the curve of Figure 3 except the vertices of the square).

If the points are statistically independent and uniformly distributed within the unit square, the ratio (heuristic TSP tour length/$\sqrt{n}$) converges almost surely as $n \to \infty$ to a constant which we have estimated to be 0.956. Similarly the ratio (weight of heuristic matching/$\sqrt{n}$) converges almost surely to $1/2 \times .956 = 0.478$. The optional additional step will not improve this assymptotic statistical behavior.

5. Comparison with other methods

Computational effort. Like the strip method, our algorithm is based on sorting, but ours is simpler and easier to code (about 20 lines of BASIC; see [2]). Furthermore, it is possible to combine the cell order of a spacefilling curve with the data structure of Iri et al. to achieve worst case linear time effort while improving the quality of their solutions. (The spacefilling curve is better than the spiral rack cell order for the same reason that the spiral is better than the serpentine cell order: the better cell orders tend to visit all of the cells in one region before continuing to a new region.)

Worst-case performance. The largest matching produced by our method is not greater that than the longest travelling salesman tour, $2\sqrt{n}$, or, with the additional step, $\sqrt{n}$. This is comparable to the $1.014\sqrt{n}$ for the spiral cell order and $.707\sqrt{n}$ for a stronger version of the strip method. The worst case ratio of heuristic to optimal matching for the strip and spiral cell order methods, as for ours, is unbounded.

Expected performance. The expected weight of the matching produced by our method on uniformly distributed points is $0.478\sqrt{n}$. This is essentially the same as for the strip method ($0.474\sqrt{n}$) and for the spiral cell method ($0.484\sqrt{n}$) [1].

Conclusions

A major advantage of our algorithm over the strip and the cell order method is its robustness. It continues to perform well under any (possibly unknown) smooth probability distribution. This makes the algorithm especially suitable solving real problems for which the true problem parameters are unknown or subject to change.

Furthermore, it produces quality solutions independent of the (normed) metric (this appears to be true for the strip and cell order methods as well). This can be useful in microprocessor implementations where floating point arithmetic might not be available.

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